Application of a Two-Dimensional Grid Solver for Three-Dimensional Problems

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Abstract

A TWO-dimensional elliptic grid solver has been applied to generate the interior and exterior body-fitted grid for a three-dimensional inlet configuration. The method uses proper forcing terms to cluster grid points near boundaries with a specified grid spacing, and allows grid lines to intersect the boundaries at a specified angle.

Contents

An integral part of the computational fluid dynamics work is the development of numerical grid-generation procedures for a body-fitted coordinate system as a practical way to apply boundary conditions. The method commonly in use is the elliptic grid solver[1-3] which solves a set of Poisson equations with appropriate right-hand-side forcing terms to achieve two main features: 1) to cluster points optimally near the boundary and in regions of high gradients in flow, and 2) to force grid lines to intersect the body surface and other computational boundaries in a nearly orthogonal fashion. In elliptic grid solvers, the quality of the grid distribution critically depends on the choice of the forcing terms.

The purpose of this synoptic is to illustrate the effective use of one such elliptic solver with suitable forcing terms in developing the computational grid for a wide variety of both internal and external three-dimensional problems. As an example, the grid generation for a three-dimensional inlet (both interior and exterior of the inlet) is presented herein. More applications can be found in Ref. 4.

The elliptic grid solver equation in the transformed (ξ,η) space can be written as

$$\alpha(x_{\xi\xi}+\phi x_\xi)-2\beta x_{\xi\eta}+\gamma(x_{\eta\eta}\psi x_\eta)=0$$
$$\alpha(y_{\xi\xi}+\phi y_\xi)-2\beta y_{\xi\eta}+\gamma(y_{\eta\eta}\psi y_\eta)=0 \tag{1}$$

where $\alpha=x_\eta^2+y_\eta^2$, $\beta=x_\xi x_\eta+y_\xi y_\eta$, and $\gamma=x_\xi^2+y_\xi^2$.

Equation (1) contains two forcing parameters: ϕ and ψ. The role of these parameters is to enforce two conditions that the grid lines have to satisfy: 1) to maintain a given ΔS grid spacing in the transverse direction η at the surface $\eta=\eta_b$, and 2) to provide a specified intersecting angle θ between the surface $(\eta=\eta_b)$ and the transverse coordinate $(\xi=\text{const})$. Solving for ϕ and ψ from Eq. (1), one gets

$$\phi=-\frac{1}{J}(y_\eta x_{\xi\xi}-x_\eta y_{\xi\xi})+\frac{2\beta}{\alpha J}(x_{\xi\eta}y_\eta-y_{\xi\eta}x_\eta)$$
$$-\frac{\gamma}{\alpha J}(y_\eta x_{\eta\eta}-x_\eta y_{\eta\eta}) \tag{2}$$

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and a similar expression for ψ. Equation (2) is valid at every grid point. An estimate for ϕ and ψ on the computational boundaries can be obtained by prescribing ΔS and θ values. From these prescriptions, the following relationships are obtained:

$$(x_\eta)_{\eta=\eta_b}=S_\eta(-x_\xi\cos\theta-y_\xi\sin\theta)/\sqrt{x_\xi^2+y_\xi^2}$$
$$(y_\eta)_{\eta=\eta_b}=S_\eta(-y_\xi\cos\theta+x_\xi\sin\theta)/\sqrt{x_\xi^2+y_\xi^2} \tag{3}$$

where $S_\eta=\Delta S/\Delta\eta$, a user-specified value. Once the grid points along the boundary $\eta=\eta_b$ are prescribed, the derivatives x_ξ, y_ξ, $x_{\xi\xi}$, and $y_{\xi\xi}$ appearing in Eq. (2) are easily evaluated. Values for $x_{\eta\eta}$ and $y_{\eta\eta}$ at $\eta=\eta_b$ are obtained using special one-sided differences[4] involving x_η and y_η, given by Eq. (3). Knowing values for all of the derivatives appearing in Eq. (2), estimates for ϕ and ψ along computational boundary grid points are first obtained. Their values at interior mesh points are then computed by interpolation. The ϕ and ψ values along boundaries are updated continuously after each relaxation cycle.

The procedure described so far generates a two-dimensional grid in the x,y plane. If the boundary, say $\eta=\eta_b$, is not confined to a two-dimensional plane, that is, along $\eta=\eta_b$, all x, y, and z are specified as shown in Fig. 1. Then, one can generate a warped computational plane that contains the boundary $\eta=\eta_b$ in the following manner:

1) First, solve for x,y from Eq. (1). Ths provides the x,y grid in the projected plane (see Fig. 1).

2) Then, solve the Laplace equation

$$z_{xx}+z_{yy}=0 \tag{4}$$

with z prescribed along the boundaries. In the transformed space, Eq. (4) becomes

$$\alpha(z_{\xi\xi}+\phi z_\xi)-2\beta z_{\xi\eta}+\gamma(z_{\eta\eta}+\psi z_\eta)=0 \tag{5}$$

Solution to step 1 above provides values of α, β, γ, ϕ, and ψ that are required in solving Eq. (5).

The grid results presented herein are obtained by solving Eqs. (1) and (5) simultaneously. The method has been effectively used to generate both the internal and external grid for a three-dimensional inlet system, and also the external grid for varieties of sharp leading-edge wing-body combinations.[4] A typical grid calculation required approximately 100 relaxation cycles to converge the residual to the 10^{-8} level.

Figure 2 shows a schematic of a three-dimensional inlet embedded in a global computational domain. The objective is to develop a body-fitted coordinate system both on the exterior and interior of the inlet. The leading edge of the inlet (1-2-3-4-5-6-7-1 in Fig. 2) is highly swept and curved. The intent is to perform a Navier-Stokes calculation for that geometry, which will require a clustered grid near the leading edge as well as near the walls of the inlet. To create a clustered body-fitted grid system, as well as a leading-edge fitted grid

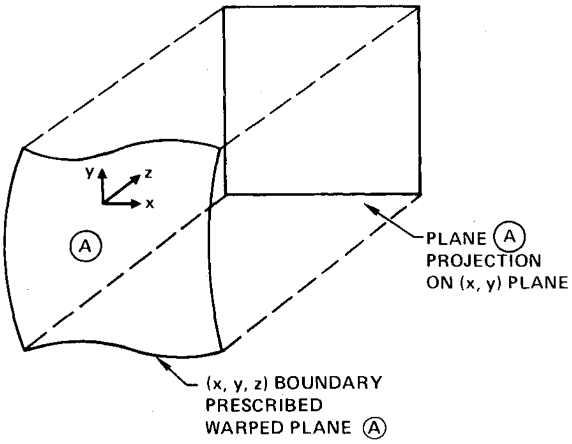

Fig. 1 Grid generation on a curved surface.

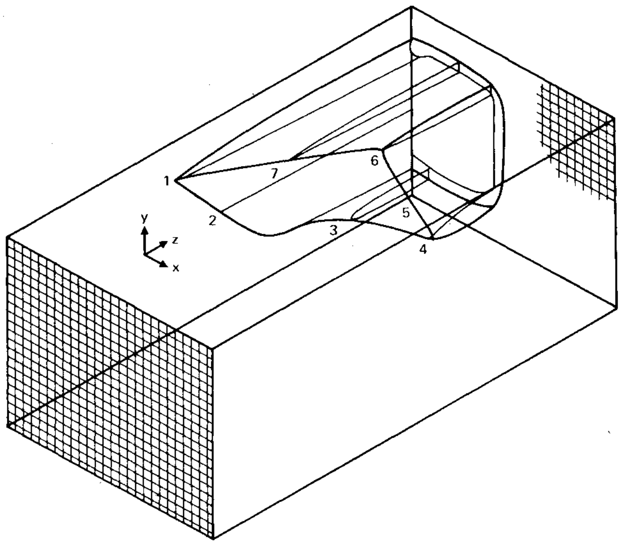

Fig. 2 Schematic of a three-dimensional inlet.

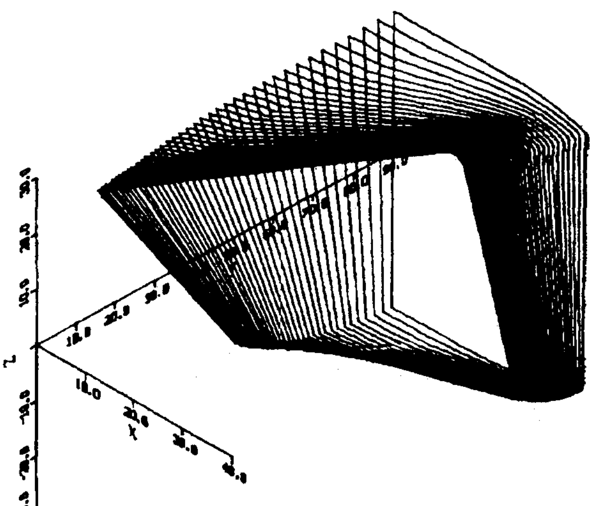

Fig. 3 Construction of inlet loops.

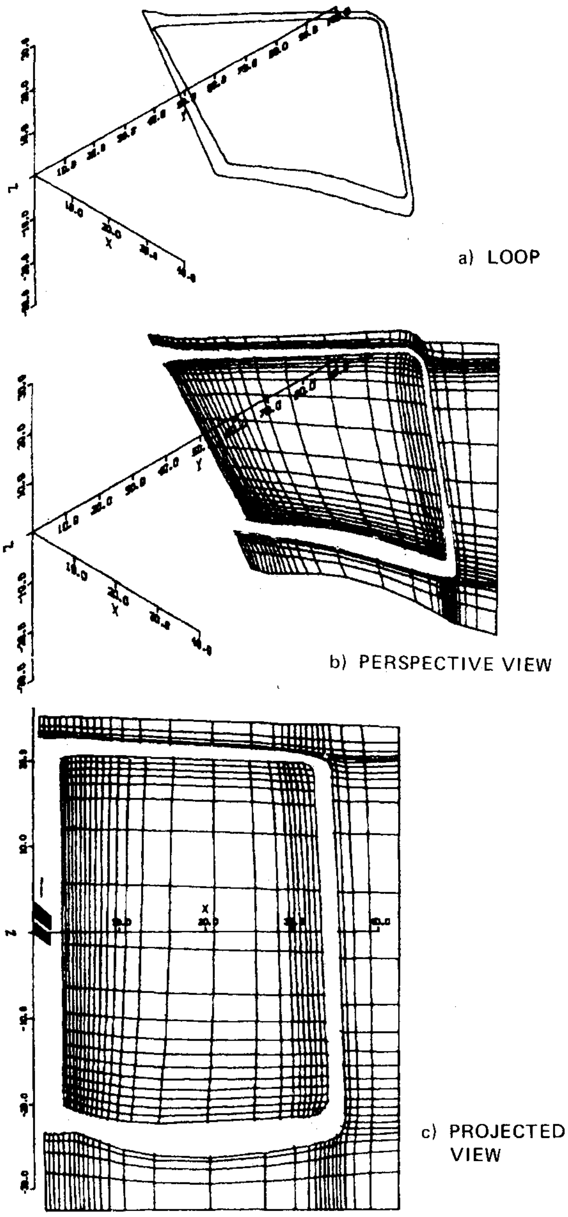

Fig. 4 Body-fitted grid system at the midstation of the inlet.

system, it was decided to gradually rotate the upstream constant z plane such that there will be a leading-edge plane containing the entire leading edge 1-2-3-4-5-6-7-1. This warped leading-edge plane would then be gradually rotated to a constant z downstream plane. In order to achieve this, the first series of loops was created starting from the leading edge to the downstream plane as shown in Fig. 3. For each loop, the inner and outer inlet wall geometry in terms of (x,y,z) was prescribed. Then, for each loop, a warped plane containing the body-fitted grid system was generated using the x,y grid solver procedure of Eq. (1), and then a "z" value at each grid point from Eq. (5).

Figure 4 shows a loop near the midsection of the inlet, a perspective view of the warped body-fitted grid surface, and a constant z projected plane showing the details of the interior and exterior grid.

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